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OPTIMAL CONTROL OF SYSTEMS WITH UNCERTAINTY.(U)
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OPTIMAL CONTROL OF SYSTEMS WITH UNCERTAINTY

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Principal Investigator: Professor W. E. Schmitendorf
Mechanical Engineering Department
Northwestern University
Evanston, Illinois 60201

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Optimal Control of Systems With Uncertainty

In the control of complex systems, uncertainties will usually occur in the mathematical description of the system. For example, the differential equations describing the system may not be known exactly or it may not be possible to make exact measurements of the state of the system. Air Force systems such as air-to-air missile encounters with an aircraft or missile guidance systems are examples of such problems. Ignoring uncertainties in the design of controllers for these systems may result in the actual system performing poorly and inaccurately. Proper methods for analyzing systems with uncertainty are needed. The research conducted under AFOSR-Grant 76-2923 has addressed the problem of the optimal control of systems with uncertainty.

Our approach to these problems is to assume that nature is perverse and will choose the uncertainty to maximize the performance index which the controller is trying to minimize. For each control there is a guaranteed performance and the optimal control is the one which achieves the best guaranteed performance. This approach leads quite naturally to the concept of minmax control. A minmax control has the appealing property of producing the best possible guaranteed performance. Unlike the stochastic approach to uncertainty, the minmax approach does not require that the statistics of the uncertainty be known. This is advantageous since the statistics of the uncertainty are often difficult to estimate. Also, a minmax control may be more easy to determine and implement than a stochastic control.

Few results are available which can be applied to obtain minmax solutions. If the problem is considered as a zero sum differential game and if this game has a saddle point solution then the minimizing control in the saddle point solution pair is also a minmax control. In this case, the theory for zero sum differential games can be applied. However, as shown by our examples, saddle point solutions seldom exist for uncertain systems and new techniques must be developed for constructing minmax solution for these problems. While the minmax approach is a natural way to treat problems with uncertainty, methods for determining minmax solutions must still be developed before this approach can be used in practice. The research undertaken has been directed toward developing such techniques and thereby aid in the design of systems with uncertainty.

While the main concern of the investigation has been with dynamic minmax problems, it was felt that it would be worthwhile to also investigate static problems. The reasons for this are two-fold. First, static problems are easier to solve than dynamic ones; yet the characteristics of the solutions of both problems have much in common. A deeper understanding of the static case is useful in the analysis of dynamic problems. Secondly, the condition for dynamic minmax problems analogous to Pontryagin's principle involves a static minmax problem and the results for static minmax problems are used in the dynamic case.

Necessary conditions and sufficient conditions for static minmax problems have been developed and are presented in [1]. The necessary conditions, in the form of a Lagrange multiplier rule, can be used to determine candidates for the solution. The sufficient conditions can be used to verify whether a candidate is indeed the solution. Since the sufficient conditions involve a strengthening of the necessary conditions, they are easy to apply once a candidate has been obtained.

In some problems the performance of the system cannot be measured by a single criterion alone, but multiple criteria are needed. The minmax results of [1] were extended to problems with multiple criteria in [2]. It is also shown there that solution candidates for the multicriteria case can be obtained by solving a related problem with a scalar criterion. This simplifies the effort needed to obtain solutions to problems with multiple criteria.

With dynamic systems, the uncertainty or disturbance may enter the system through the state equations or through the initial conditions. First consider problems with time-varying uncertainty in the state equations. These problems arise when it is not possible to obtain an exact model of the system. Often the analysis is carried out by neglecting the uncertainty. However, this may be too idealized for the analysis to be valid and the actual system may not perform well. Then the analysis must take into account the fact that the model is not exact and the minmax solution concept is an attractive way to treat the uncertainty since it assures the best possible guaranteed performance. In practice, nature will probably not be so perverse as to choose the disturbance to maximize the performance index and the system will perform better than predicted. However, if a control not having the minmax property is used, the system may perform decidedly worse than expected. Thus a control having the minmax property should be used in the design of systems with uncertainty when there is no a priori knowledge of the value of the uncertainty.

We have developed a sufficient condition which the minmax control must satisfy [3]. This condition also leads to a method for constructing a minmax control. It has been shown that the minmax control can be obtained by solving a related optimal control problem <u>without</u> uncertainty. Thus the well-developed techniques from deterministic optimal control theory can be used to solve problems with uncertainty via this related problem. This is thought to be a significant step in the direction of obtaining methods which can be readily used to solve problems with time varying uncertainty in the state equation.

In [4], problems with uncertain initial conditions are treated. In these problems, the exact value of the initial state is not known. Instead, only an inexact measurement is available. This is often the case in realistic situations where, for example, due to hardware limitations, position and velocity cannot be measured exactly. All that is available is the measured values of those quantities which equal the true values plus or minus some error. Our results were obtained by using a transformation which transforms the original problem with uncertainty in the initial state to one with the initial state known but with parameter uncertainty in the state equation. The latter problem is simpler to solve. It can be shown that the solution of the new problem is a solution to the original one with uncertain initial condition. Through this observation we are able to present a constructive technique for finding the minmax control and also a sufficient condition which can be used to verify sufficiency. As a by-product, our techniques can also be used to solve some problems where there is parameter uncertainty in the state equation (rather than time-varying uncertainty). Parameter uncertainty often occurs in the system model when there is a lack of experimental data so that the exact values of the parameters in the model are unknown.

In summary, our investigation of static minmax problems has produced necessary conditions and sufficient conditions which can be used to solve such problems. This has also led to a clearer understanding of the nature of minmax problems. For dynamic problems with uncertainty in the state equations or in the initial conditions, we derived sufficient conditions which the minmax control must satisfy and, using these conditions, constructive techniques were developed which can be used to generate minmax solutions. These methods are now available for solving minmax problems and can be used to analyze problems where there is uncertainty in the model or in the measurement of the initial state. They will aid in the design of systems where exact models of the system are not available or where exact measurements of the state of the system cannot be obtained.

Finally, a detailed description of our results can be found in the papers which have resulted from our research. These papers are included in the Appendix.

References (Papers resulting from the research sponsored by AFOSR under Grant AFOSR 76-2923)

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2. "Static multicriteria problems: Necessary conditions and sufficient conditions," Proceedings of the IFAC Symposium on Large Scale Systems, Udine, Italy, June 16-20, 1976.

3. "A sufficient condition for minmax control of systems with uncertainty in the state equations," IEEE Trans. Automatic Control, Vol. AC-21, No. 4, 1976. Also in Proceedings 1976 Joint Automatic Control Conference, Lafayette, Indiana, July 1976.

4. "Minmax control of systems with uncertainty in the initial state and in the state equations," Proceedings 1976 Conference on Decision and Control, Clearwater Beach, Florida, December 1976 (to appear). Also submitted for publication to IEEE Trans. Automatic Control.

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This investigation of static minmax problems has produced necessary and sufficient conditions which can be used to solve such problems. This has also led to a clearer understanding of the nature of minmax problems for dynamic problems with uncertainty in the state equations or in the initial conditions, sufficient conditions were derived which the minmax control must satisfy and, using these conditions, constructive techniques were developed which can be used to generate minmax solutions. These methods are now available for solving minmax problems and can be used to analyze problems where there is uncertainty in the model or in the measurement of →

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